

# (1, 2) -Vertex Domination in Fuzzy Graphs

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**ABSTRACT:** A (1,2) - dominating set in a fuzzy graph  $G = (V,E)$  is a set  $S$  having the property that for every vertex  $v$  in  $V - S$ , There is at least one vertex in  $S$  at distance 1 from  $v$  and a second vertex in  $S$  at distance almost 2 from  $v$ . The minimum cardinality of (1,2) - dominating set in fuzzy graph  $G$  is called the (1,2) - domination number in  $G$  and we denote it by  $\gamma(1,2)$ . We introduce (1,2) - domination number of a fuzzy graph and obtain some interesting results for the new parameter in fuzzy graph.

**KEYWORDS:** Fuzzy graphs, (1,2) - dominating set in a fuzzy graph , (1,2) - domination number in a fuzzy graph.

## 1. INTRODUCTION

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths cycles and connectedness are introduced by Rosenfeld [1]. Bhattacharya[2] has established some connectivity regarding fuzzy cut node and fuzzy bridges. The concept of domination in fuzzy graphs are introduced by A.Somasudaram and S.Somasundaram[6] in 1998. The concept of (1, 2) domination in fuzzy graphs are introduced by N. Murugesan & Deepa.S.Nair in [ 4].In this paper, We analyze bounds on (1, 2) domination in fuzzy graphs.

## II. PRELIMINARIES

### Definition 2.1

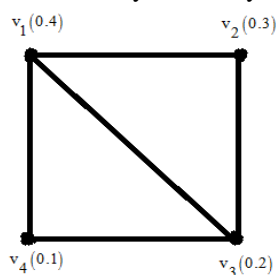
A fuzzy subset of a nonempty set  $V$  is mapping  $\sigma: V \rightarrow [0, 1]$  and A fuzzy relation on  $V$  is fuzzy subset of  $V \times V$ . A fuzzy graph is a pair  $G: (\sigma, \mu)$  where  $\sigma$  is a fuzzy subset of a set  $V$  and  $\mu$  is a fuzzy relation on  $\sigma$ , where  $\mu(u,v) \leq \sigma(u) \wedge \sigma(v) \forall u,v \in V$

### Definition 2.2

A fuzzy graph  $G=(\sigma, \mu)$  is a strong fuzzy graph if  $\mu(u,v) = \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$  and is a complete fuzzy graph if  $\mu(u,v) = \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$ .The complement of a fuzzy graph  $G=(\sigma, \mu)$  is a fuzzy graph  $\bar{G} = (\bar{\sigma}, \bar{\mu})$  where  $\bar{\sigma} = \sigma$  and  $\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$  for all  $u, v \in V$ . we denote a cycle on  $P$  vertices by  $C_p$ , a path by  $P_p$  a complete graph on  $P$  vertices by  $K_p$ .

### Definition 2.3

Let  $G=(\sigma, \mu)$  be a fuzzy graph. Then  $D \subseteq V$  is said to be a fuzzy dominating set of  $G$  if for every  $v \in V-D$ , There exists  $u$  in  $D$  such that  $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ . The minimum scalar cardinality of  $D$  is called the fuzzy dominating number and is denoted by  $\gamma(G)$ . Note that scalar cardinality of a fuzzy subset  $D$  of  $V$  is  $|D| = \sum_{v \in V} \sigma(v)$



$$D = \{v_1\}, \gamma(G) = 0.4$$

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### Definition 2.4

A dominating set  $D$  of a fuzzy graph  $G = (\sigma, \mu)$  is connected dominating set if the induced fuzzy sub graph  $\langle D \rangle$  is connected. The minimum cardinality of a connected dominating set of  $G$  is called the connected domination number of  $G$  and is denoted by  $\gamma_c(G)$

### III. (1, 2) - DOMINATION IN FUZZY GRAPHS.

In this section we present few elementary bounds on (1, 2)- vertex domination in fuzzy graphs. The (1, 2) - domination is based on vertex. So (1, 2)- vertex domination is denoted by (1, 2)- domination and corresponding some results.

### Definition 3.1

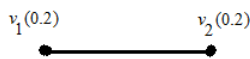
A (1,2) - dominating set in a fuzzy graph  $G = (V,E)$  is a set  $S$  having the property that for every vertex  $v$  in  $V - S$  There is at least one vertex in  $S$  at distance 1 from  $v$  and a second vertex in  $S$  at distance almost 2 from  $v$ . The minimum cardinality of (1,2) - dominating set in fuzzy graph  $G$  is called the (1,2) - domination number in  $G$  and we denote it by  $\gamma(1,2)$ .

### Definition 3.2

A harmonius fuzzy graph is a connected labeled fuzzy graph with  $n$  graph edges in which all fuzzy graph vertices can be labeled with distinct integers (mod  $n$ ) so that the sums of the pairs of numbers at the ends of each graph edge are also distinct (mod  $n$ ). The ladder fuzzy graph ( $L_n$ ) and wheel fuzzy graph ( $W_n$ ) are harmonius. This fuzzy graph looks like a ladder, having two rails and  $n$  rungs between them. A wheel fuzzy graph  $W_n$  of order  $n$ , contains a cycle of order  $n-1$ , and for which every graph vertex in the cycle is connected to one other fuzzy graph vertex.

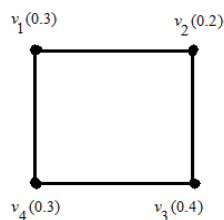
In this section we consider ladder fuzzy graphs of order up to 6 and find out their domination number and (1,2) - domination number.

For  $n = 1$ ,



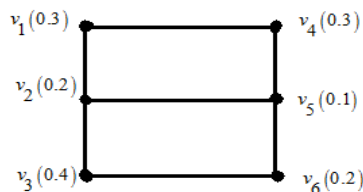
This is a fuzzy graph of order 2. (1,2) - domination number is defined for fuzzy graphs of order at least 3.

For  $n = 2$ ,



$\{v_3, v_4\}$  is a dominating set and also a (1,2) - dominating set.  $\gamma(1,2) = 0.7$  and  $\gamma = 0.7$

For  $n = 3$ ,



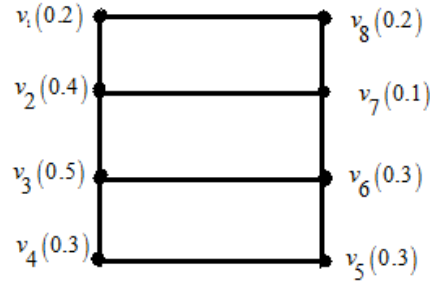
$\{v_1, v_2, v_3\}$  is a (1,2) - dominating set.  $\{v_2, v_5\}$  is a dominating set,  $\gamma(1,2) = 0.9$  and  $\gamma = 0.3$

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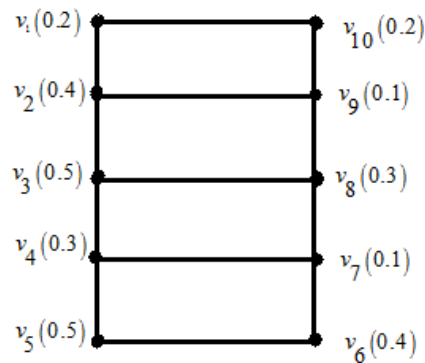
Vol. 5, Issue 9, September 2016

For n = 4,



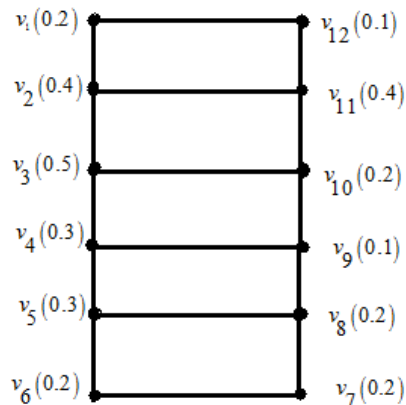
$\{v_1, v_2, v_3, v_4\}$  is a (1,2) - dominating set.  $\{v_1, v_3, v_5, v_7\}$  is a dominating set.  $\gamma(1,2) = 1.4$  and  $\gamma = 1.1$

For n = 5,



$\{v_1, v_2, v_3, v_4, v_5\}$  is a (1,2) - dominating set.  $\{v_1, v_3, v_6, v_8\}$  is a dominating set.  $\gamma(1,2) = 1.9$  and  $\gamma = 1.4$

For n = 6,



$\{v_1, v_2, v_3, v_4, v_5, v_6\}$  is a (1,2) - dominating set.  $\{v_1, v_3, v_5, v_7, v_9, v_{11}\}$  is a dominating set,  $\gamma(1,2) = 1.9$  and  $\gamma = 1.5$

From the above examples we have the following theorems.

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Vol. 5, Issue 9, September 2016

### Theorem 3.3

For a ladder fuzzy graph  $L_n$ , (1,2)- dominating vertices is  $n$ .

#### Proof

For a ladder fuzzy graph  $L_n$ , there are  $3n-2$  edges and  $2n$  vertices. Also there are  $n$  vertices in both the rails. Suppose a vertex  $v_1$  in the first rail is adjacent to a vertex  $u_1$  in the second rail. Then all the remaining vertices in the first rail will be at distance greater than 1 from  $u_1$ . So to form a (1,2) - dominating set we have to include all the vertices in one rail. So the (1,2) - dominating vertices is  $n$ .

### Theorem 3.4

For a ladder fuzzy graph  $L_n$  with  $n$  even, dominating vertices is  $n$ .

#### Proof

Each  $L_n$  has  $3n-2$  edges and  $2n$  vertices. If  $n$  is even, the vertices in the inner rungs, that is  $n/2$  rungs can form a dominating set. So the number of vertices in the dominating set will be  $n$ , since each rung contains two vertices.

### Theorem 3.5

For a ladder fuzzy graph  $L_n$  with  $n$  odd, dominating vertices is  $n-1$ .

#### Proof

Each  $L_n$  has  $3n-2$  edges and  $2n$  vertices. Since  $n$  is odd, the vertices in the middle rung will be at equal distance from the vertices in the outer rungs. So if we take the two vertices of the middle rung and one vertex each from the alternate rungs, that set will form a dominating set. So since there are  $n$  rungs, the set will consist of  $n-1$  vertices. So the dominating vertices is  $n-1$ .

### Theorem 3.6

In a fuzzy graph  $G$ , number of dominating vertices is less than or equal to number of (1,2)- dominating vertices.

#### Proof

Let  $G$  be a fuzzy graph and  $D$  be its dominating set. Then every vertex in  $V-D$  is adjacent to a vertex in  $D$ . That is, in  $D$ , for every vertex  $u$ , there is a vertex which is at distance 1 from  $u$ . But it is not necessary that there is a second vertex at distance at most 2 from  $u$ . So if we find a (1,2)- dominating set, it will contain more vertices or at least equal number of vertices than the dominating set. So the number of dominating vertices is less than or equal to number of (1,2)- dominating vertices.

This is true for ladder fuzzy graphs also.

### Theorem 3.7

For a ladder fuzzy graph  $L_n$  with  $n$  even, the number of dominating vertices and number of (1,2) - dominating vertices are equal.

#### Proof

In a ladder fuzzy graph, there are  $2n$  vertices and  $3n-2$  edges. The  $n$  vertices in one rail form a (1,2) - dominating set. If  $n$  is even the number of inner rungs will be  $n/2$  even. And the vertices of these inner rungs form a dominating set. Since each rung contains 2 vertices, the dominating set will consist of  $n$  vertices. Hence the number of dominating vertices and the number of (1,2) -dominating vertices are equal.

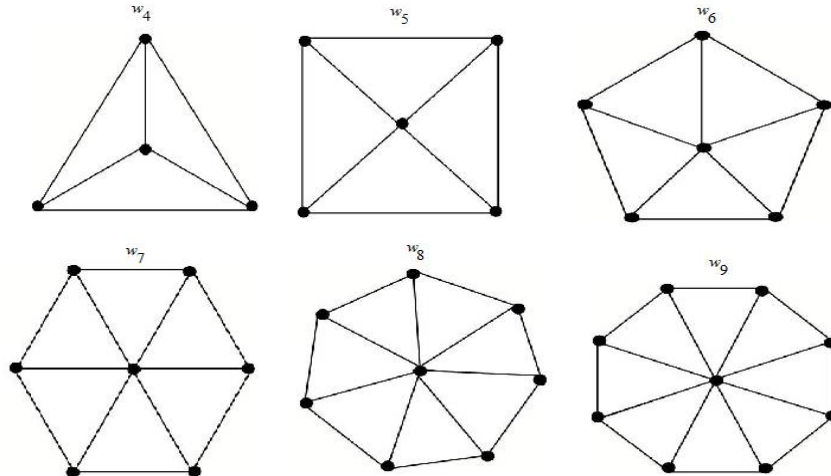
### Theorem 3.8

For  $n$  odd, the number of dominating vertices of a ladder fuzzy graph  $L_n$  is less than the number of (1,2) -dominating vertices

#### Proof

For a ladder fuzzy graph with  $n$  odd, the number of inner rungs will be  $(n-2)/2$ , odd. The vertices of the middle rung and one vertex each from the alternate rungs will form a dominating set. So altogether we will get  $(n-1)$  vertices. That is, the dominating vertices is  $(n-1)$ . But the (1,2) - dominating vertices is  $n$ . Hence the number of dominating vertices of a ladder fuzzy graph  $L_n$  is less than the number of (1,2) -dominating vertices

Consider the following wheel fuzzy graphs.



**Theorem 3.9**

For a wheel fuzzy graph, the dominating vertex is 1.

**Proof**

In a wheel fuzzy graph it contains a cycle of order  $n - 1$  every fuzzy graph vertex in the cycle is connected to one other fuzzy graph vertex. In a wheel fuzzy graph  $W_n$ , So that vertex is adjacent to all other vertices. Hence the dominating vertex is 1.

**Theorem 3.10**

For a wheel fuzzy graph  $W_n$ , (1,2) -dominating vertex is 2.

**Proof**

The dominating set of a wheel fuzzy graph consists of only one vertex. By the definition of (1,2) - dominating set, it should contain at least two vertices. So if we take the central vertex and any one of the vertex from the cycle, that will form a (1,2) - dominating set. The cardinality of the (1,2) - dominating is 2.

**IV. CONCLUSION**

(1,2) –vertex domination in fuzzy graph is defined. Theorems related to this concept are derived and the relation between domination number in fuzzy graph and (1,2) - vertex domination in fuzzy graphs are established.

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